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Event-triggered cooperative target tracking in wireless sensor networks

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Abstract Since the issues of low communication bandwidth supply and limited battery capacity are very crucial for wireless sensor networks, this paper focuses on the problem of event-triggered cooperative target tracking based on set-membership information filtering. We study some fundamental properties of the set-membership information filter with multiple sensor measurements. First, a sufficient condition is derived for the set-membership information filter, under which the boundedness of the outer ellipsoidal approximation set of the estimation means is guaranteed. Second, the equivalence property between the parallel and sequential versions of the set-membership information filter is presented. Finally, the results are applied to a 1D event-triggered target tracking scenario in which the negative information is exploited in the sense that the measurements that do not satisfy the triggering conditions are modelled as set-membership measurements. The tracking performance of the proposed method is validated with extensive Monte Carlo simulations.

1. Introduction

In recent years, cooperative target tracking in wireless sensor networks (WSNs) has a wide range of applications in the field of intelligence, surveillance and reconnaissance (ISR).[1–3] The application of distributed WSNs provides a competent method for battlefield information collection and a robust scheme for moving target tracking[4,5] in a complex and interference-rich environment. However, the limited network resources in terms of energy and communication bandwidth set a constraint on the ability of WSNs.[6–8] Since the sensors are dispersed via airdrop or cannon fire in a lot of practical scenarios,[9] their batteries are difficult to be recharged or replaced. Thus, an energy-saving tracking strategy is demanded to conserve network energy and extend network life.

For wireless ad hoc networks, the energy consumption depends heavily on the wireless communication.[10] Since WSNs face stringent energy limits, forcing all sensors to communicate with the fusion center (FC) at full rate is apparently not

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affordable. Therefore, an energy-saving strategy in our tracking architecture is to reduce the communication frequency while guarantee an acceptable tracking accuracy.[11,12] Event-triggered estimation is a promising alternative to fulfill such imperative requirement.[13] In contrast to sending the measurements periodically, the local sensors employ various event sampling strategies to determine whether to send up-to-date measurements to the remote FC, such as Matched Sampling,[14] Integral Sampling,[15] Send-on-Delta (SOD)[16] and Variance-based Triggering.[17] As a result, the event-triggered strategy is to estimate the state based on the intermittent observations[18] that satisfy the triggering conditions, but this cannot be applied straightforwardly in target tracking applications because the improper design of triggering thresholds might result in the loss of track[19,20] as the FC receives no observation at a number of consecutive time steps.

In cases that the sensors determine not to send their current measurements, some "negative information"[21] is implicitly available to the remote FC without additional communication. For example, when the SOD method is employed, no measurement transmission from one specific sensor implies that the value of the current measurement does not deviate too much from the value of the last transmitted measurement,[22] which suggests that although the exact value of the current measurement is unknown to the FC, it lies in a set formulated by the SOD triggering condition. In order to better exploit this "negative information", as well as to integrate it into the Bayesian state estimation framework, several set-membership state estimators[23–25] have been proposed, where the uncertainty of the negative information is modelled as a set of Gaussian densities. In Ref.[26], a generalization of the standard Kalman filter is developed to solve the problem of set-membership measurement for the single sensor. The work in Ref.[27] reveals that the information form of set-membership filtering consists of advantageous properties especially when multiple set-membership measurements are received. Despite the great deal of effort that has been dedicated to it, several problems remain open, which are of significant importance in studying the set-membership filter and its application to the problem of event-triggered cooperative target tracking in WSNs. The first issue is that the boundedness of the set of the estimation means has not been investigated systematically when the set-membership measurement is considered.[27,28] Another issue is to explore the equivalence between the parallel and recursive implementations of the set-membership information filter, and particularly since the Minkowski sum of ellipsoids[29] might not be an ellipsoid, some approximation of the exact estimation result is inevitable.

In this paper, we study the boundedness of the set of the estimation means with the information form, where a sufficient condition is proved, under which the outer ellipsoidal approximation set is asymptotically bounded. We also present the equivalence between the parallel and sequential set-membership information filters. Finally, the set-membership information filter is applied to the problem of event-triggered cooperative target tracking, and the performance of the proposed tracking strategy is further validated with extensive Monte Carlo simulations.

2. Problem formulation

For a random vector $\boldsymbol{x} \in \mathbf{R}^n$, we use $E(\boldsymbol{x})$ and $\mathrm{Cov}(\boldsymbol{x})$ to denote its mean and covariance respectively. For a matrix $\boldsymbol{A} \in \mathbf{R}^{n\times n}$, we define $\mathrm{tr}(\boldsymbol{A})$ as its trace. Given $\boldsymbol{S} \in \mathbf{R}^{n\times n} > \mathbf{0}$, i.e., $\boldsymbol{S}$ is positive definite, an ellipsoidal set $\mathcal{X} = \varepsilon(\boldsymbol{c}, \boldsymbol{S})$ is represented as

$$\mathcal{X} \triangleq \varepsilon(\boldsymbol{c}, \boldsymbol{S}) = \left\{ \boldsymbol{x} \in \mathbf{R}^n | (\boldsymbol{x}-\boldsymbol{c})^{\mathrm{T}} \boldsymbol{S}^{-1} (\boldsymbol{x}-\boldsymbol{c}) \leqslant 1 \right\} \tag{1}$$

For two ellipsoidal sets $\mathcal{X}$ and $\mathcal{Y}$, let $\mathcal{X} \oplus \mathcal{Y}$ denotes their Minkowski sum, namely $\mathcal{X} \oplus \mathcal{Y} \triangleq \{\boldsymbol{x}+\boldsymbol{y} | \boldsymbol{x} \in \mathcal{X}, \boldsymbol{y} \in \mathcal{Y}\}$, and we have $\sum_{i=1}^{N} \mathcal{X}_i \triangleq \mathcal{X}_1 \oplus \mathcal{X}_2 \oplus \cdots \oplus \mathcal{X}_N$.

We consider a linear time-invariant dynamic system that evolves in discrete time and is perturbed by Gaussian white noise as follows:

$$\boldsymbol{x}(k) = \boldsymbol{F}\boldsymbol{x}(k-1) + \boldsymbol{w}(k) \tag{2}$$

where $\boldsymbol{x} \in \mathbf{R}^n$; $\boldsymbol{w} \sim N(\mathbf{0}, \boldsymbol{Q})$; $\boldsymbol{F}$ is the transition matrix of the dynamic model; and $\boldsymbol{Q}$ is the covariance of the process noise. We assume that $(\boldsymbol{F}, \boldsymbol{Q})$ is stabilizable.[30] The state $\boldsymbol{x}$ is measured with N sensors as

$$\boldsymbol{z}_i(k) = \boldsymbol{H}_i \boldsymbol{x}(k) + \boldsymbol{v}_i(k) \quad \text{for} \quad i = 1, 2, \ldots, N \tag{3}$$

where $\boldsymbol{v}_i \sim N(\mathbf{0}, \boldsymbol{R}_i)$ denotes the measurement noise, $\boldsymbol{R}_i$ the covariance of the measurement noise for the ith sensor; N is the number of the sensors; $\boldsymbol{H}_i$ is the measurement model matrix. We also assume that $(\boldsymbol{F}, \boldsymbol{H})$ is detectable,[30] where $\boldsymbol{H} = [\boldsymbol{H}_1, \boldsymbol{H}_2, \ldots, \boldsymbol{H}_N]^{\mathrm{T}}$ and $\boldsymbol{R} = \mathrm{diag}(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N)$. In addition to the stochastic measurement noise $\boldsymbol{v}_i$, we further consider that the obtained measurement consists of an unknown but bounded error $\boldsymbol{e}_i(k)$, namely

$$\hat{\boldsymbol{z}}_i(k) = \boldsymbol{H}_i \boldsymbol{x}(k) + \boldsymbol{v}_i(k) + \boldsymbol{e}_i(k) = \boldsymbol{z}_i(k) + \boldsymbol{e}_i(k) \tag{4}$$

where the uncertainty of $\boldsymbol{e}_i(k)$ is confined to an ellipsoidal set as

$$\boldsymbol{e}_i(k) \in \varepsilon(\mathbf{0}, \boldsymbol{S}_{\boldsymbol{e}_i(k)}) \tag{5}$$

With respect to the fact that the uniqueness of the measurement $\hat{\boldsymbol{z}}_i$ cannot be maintained due to the uncertainty of $\boldsymbol{e}_i(k)$, the set-membership[31] measurement $\mathcal{Z}_i$ will be used as a replacement:

$$\mathcal{Z}_i(k) = \{\hat{\boldsymbol{z}}_i(k) - \boldsymbol{e}_i(k) | \boldsymbol{e}_i(k) \in \varepsilon(\mathbf{0}, \boldsymbol{S}_{\boldsymbol{e}_i(k)})\} = \varepsilon(\hat{\boldsymbol{z}}_i(k), \boldsymbol{S}_{\boldsymbol{e}_i(k)}) \tag{6}$$

To cope with the set-membership uncertainty of $\mathcal{Z}_i$, one feasible manner is to model the uncertainty by a set of Gaussian densities, which gives rise to the set-membership Kalman filter[32] and set-membership information filter.[27] We know that the standard information filter embodies an algebraic reformulation of the Kalman filter, which provides an easier update phase for the distributed estimation architecture by estimating the information about the state rather than the state itself.[33] More exactly, the information state

$$\boldsymbol{y} = \boldsymbol{P}^{-1}\hat{\boldsymbol{x}} \tag{7}$$

where $\boldsymbol{P}$ is estimation covariance matrix and $\hat{\boldsymbol{x}}$ the estimation mean; and the information matrix

$$\boldsymbol{Y} = \boldsymbol{P}^{-1} \tag{8}$$

are the quantities to be calculated at the prediction and update steps. In the presence of additional set-membership uncertainties, an ellipsoidal set $\varepsilon(\hat{\boldsymbol{x}}, \boldsymbol{S}_{\hat{\boldsymbol{x}}})$ of estimation means has to be processed in its information form, which is obtained by an affine transformation[34]:

$$\mathcal{Y} = \boldsymbol{P}^{-1}\varepsilon(\hat{\boldsymbol{x}}, \boldsymbol{S}_{\hat{\boldsymbol{x}}}) = \varepsilon\left(\boldsymbol{P}^{-1}\hat{\boldsymbol{x}}, \boldsymbol{P}^{-1}\boldsymbol{S}_{\hat{\boldsymbol{x}}}(\boldsymbol{P}^{-1})^{\mathrm{T}}\right) = \varepsilon(\boldsymbol{y}, \boldsymbol{S}_{\mathcal{Y}}) \tag{9}$$

and thus an ellipsoidal set including all possible information states is produced.

For conventional information filter, the measurement of the estimation accuracy of the state is fully represented as the information matrix $\boldsymbol{Y}$, while for set-membership information filter, simultaneous consideration of stochastic and set-membership estimation uncertainty should be introduced,[26] where the confidence in stochastic uncertainty is still quantified as $\boldsymbol{Y}$, and the confidence in set-membership uncertainty is quantified as the trace of $\boldsymbol{S}_{\mathcal{Y}}$.

The information matrix $\boldsymbol{Y}$ converges to one unique solution to a Riccati equation[35] as time iterates, provided that $(\boldsymbol{F}, \boldsymbol{Q})$ is stabilizable and $(\boldsymbol{F}, \boldsymbol{H})$ is detectable. However, the asymptotic behavior of the confidence in the set-membership uncertainty, i.e., $\mathrm{tr}(\boldsymbol{S}_{\mathcal{Y}})$, has not been investigated theoretically yet. Besides, since the Minkowski sum of ellipsoidal sets is difficult to calculate analytically,[24] the exact set of estimation means can only be approximated with an outer ellipsoidal set $\hat{\mathcal{Y}}$. In this regard, the boundedness property of $\mathrm{tr}(\boldsymbol{S}_{\hat{\mathcal{Y}}})$ is of importance as well since the exact set is contained in the outer approximation set.

Based on the presented notations, we are now ready to introduce the problems to be solved in this work:

(1) Investigate the asymptotic boundedness of the trace of the set $\hat{\mathcal{Y}}$.
(2) Explore the equivalence between the parallel and sequential set-membership information filters.
(3) Apply the set-membership information filter to the problem of event-triggered cooperative target tracking in WSNs.

3. Asymptotic boundedness of outer ellipsoidal set

In this section, we analyze the boundedness of the outer ellipsoidal approximation set $\hat{\mathcal{Y}}$. Here we define that an ellipsoidal set $\mathcal{X} = \varepsilon(\boldsymbol{c}, \boldsymbol{S})$ is bounded if $\mathrm{tr}(\boldsymbol{S})$ is bounded. The set-membership information filter is conceptualized as two distinct phases: "Prediction" and "Update". The prediction phase produces a set of priori estimates of the state with the following linear transformation:

$$\mathcal{Y}(k|k-1) = \boldsymbol{L}(k)\boldsymbol{F}^{-\mathrm{T}}\hat{\mathcal{Y}}(k-1|k-1) \tag{10}$$

where

$$\begin{cases} \boldsymbol{L}(k) = \boldsymbol{I} - \boldsymbol{C}(k) \\ \boldsymbol{C}(k) = \boldsymbol{M}(k)\left(\boldsymbol{M}(k) + \boldsymbol{Q}^{-1}\right)^{-1} \\ \boldsymbol{M}(k) = \boldsymbol{F}^{-\mathrm{T}}\boldsymbol{Y}(k-1|k-1)\boldsymbol{F}^{-1} \end{cases} \tag{11}$$

In the update phase, the priori set with the information form is updated with the set-membership measurements by the operation of Minkowski sum[27] as

$$\begin{aligned} \mathcal{Y}(k|k) &= \mathcal{Y}(k|k-1) \oplus \mathcal{I}_1(k) \oplus \mathcal{I}_2(k) \oplus \cdots \oplus \mathcal{I}_N(k) \\ &= \mathcal{Y}(k|k-1) \oplus \sum_{i=1}^{N} \mathcal{I}_i(k) \end{aligned} \tag{12}$$

where

$$\begin{aligned} \mathcal{I}_i(k) &= \boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{R}_i^{-1}\mathcal{Z}_i(k) = \boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{R}_i^{-1}\varepsilon(\hat{z}_i(k), \boldsymbol{S}_{e_i(k)}) \\ &= \varepsilon\left(\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{R}_i^{-1}\hat{z}_i(k), \boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{R}_i^{-1}\boldsymbol{S}_{e_i(k)}\boldsymbol{R}_i^{-\mathrm{T}}\boldsymbol{H}_i\right) \\ &= \varepsilon\left(\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{R}_i^{-1}\hat{z}_i(k), \boldsymbol{S}_{\mathcal{I}_i(k)}\right) \end{aligned} \tag{13}$$

As for the calculation of information matrix, it follows that of the standard information filter:

$$\begin{cases} \boldsymbol{Y}(k|k-1) = \boldsymbol{L}(k)\boldsymbol{M}(k)\boldsymbol{L}^{\mathrm{T}}(k) + \boldsymbol{C}(k)\boldsymbol{Q}^{-1}\boldsymbol{C}^{\mathrm{T}}(k) \\ \boldsymbol{Y}(k|k) = \boldsymbol{Y}(k|k-1) + \sum_{i=1}^{N}\boldsymbol{E}_i(k) \end{cases} \tag{14}$$

where

$$\boldsymbol{E}_i(k) = \boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{R}_i^{-1}\boldsymbol{H}_i \tag{15}$$

From Eqs. (14) and (15), it can be inferred that the set of estimation means shares the same covariance, since the estimation error covariance is independent of $\mathcal{Z}_i$ in the set-membership information filtering framework.

Due to the fact that the Minkowski sum of ellipsoids in Eq. (12) does not yield an ellipsoid, an outer ellipsoidal approximation is calculated according to the following result[34]:

$$\sum_{i=1}^{N}\varepsilon(\boldsymbol{c}_i, \boldsymbol{S}_i) \subseteq \varepsilon(\boldsymbol{c}, \boldsymbol{S}) \tag{16}$$

with

$$\begin{cases} \boldsymbol{c} = \sum_{i=1}^{N}\boldsymbol{c}_i \\ \boldsymbol{S} = \left(\sum_{i=1}^{N}p_i\right)\cdot\left(\sum_{i=1}^{N}p_i^{-1}\boldsymbol{S}_i\right) \end{cases} \tag{17}$$

for each set of $p_i > 0$ $(i = 1, 2, \ldots, N)$. Consequently $\mathcal{Y}(k|k)$ in Eq. (12) can be externally approximated as

$$\mathcal{Y}(k|k) = \mathcal{Y}(k|k-1) \oplus \sum_{i=1}^{N}\mathcal{I}_i(k) \subseteq \hat{\mathcal{Y}}(k|k) \tag{18}$$

Now we are in the position to introduce the result on the asymptotic boundedness property of the outer ellipsoidal approximation set $\hat{\mathcal{Y}}$.

Theorem 1. *Assume that* $(\boldsymbol{F}, \boldsymbol{Q})$ *is stabilizable and* $(\boldsymbol{F}, \boldsymbol{H})$ *is detectable. Let* $\bar{\boldsymbol{L}} = \lim_{k\to\infty}\boldsymbol{L}(k)$ *and* $\bar{\boldsymbol{A}} = \bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}}$. *If* $\boldsymbol{0} \leqslant (\bar{\boldsymbol{A}})^{\mathrm{T}}\bar{\boldsymbol{A}} < \boldsymbol{I}$, *then the set* $\hat{\mathcal{Y}}$ *is asymptotically bounded for all measurement sets* $\mathcal{Z}_i$ *with bounded sizes* $\mathrm{tr}(\boldsymbol{S}_{e_i(k)})$.

Appendix A shows the proof of Theorem 1. In the following, we present an example to show the boundedness of the outer ellipsoidal set.

Example 1. We consider a system in Eq. (2) with $\boldsymbol{F} = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.8 \end{bmatrix}$ and $\boldsymbol{Q} = \begin{bmatrix} 0.333 & 0.5 \\ 0.5 & 0.1 \end{bmatrix}$ measured by four sensors described in Eq. (4) with $\boldsymbol{H}_i = [1, 0]$ for $i = 1, 2, 3, 4$, $R_1 = 1$, $R_2 = 2$, $R_3 = 3$, $R_4 = 4$ and $S_{e_1} = 2$, $S_{e_2} = 3$, $S_{e_3} = 6$, $S_{e_4} = 7$, where $(\boldsymbol{F}, \boldsymbol{Q})$ is stabilizable and $(\boldsymbol{F}, \boldsymbol{H})$ is detectable. It can be further proved that $\boldsymbol{0} \leqslant (\bar{\boldsymbol{A}})^{\mathrm{T}}\bar{\boldsymbol{A}} < \boldsymbol{I}$ which implies that the boundedness of $\mathrm{tr}(\boldsymbol{S}_y)$ is guaranteed according to Theorem 1. The center of the set of the estimation errors and its bounds are shown in Fig. 1 for the first and second states respectively. It is observed that the bounds converge over time.

4. Equivalence between parallel and sequential set-membership information filters

In this section, the purpose is to examine the equivalence between two versions of set-membership information filter for a multi-sensor system. As shown in Figs. 2 and 3, the major difference between them lies in the manner in which they process the set-membership measurements at the update step. The first filter which has been introduced in Eq. (12) processes all the set-membership measurements in parallel, while the second filter updates the set-membership measurements in a recursive manner, which offers the benefit that the matrix $\boldsymbol{S}_{\hat{\mathcal{Y}}(k|k)}$ in Eq. (A6) does not need to be computed at once, but can be computed sensor by sensor.

To aid the analysis, we first suppose that the parallel rule is as follows:

$$\mathcal{Y}(k|k-1) \oplus \sum_{i=1}^{n} \mathcal{I}_i(k) \subseteq \hat{\mathcal{Y}}^{\mathrm{p}}_{1,2,\ldots,n}(k|k) \triangleq \varepsilon\left(\hat{\boldsymbol{y}}^{\mathrm{p}}_{1,2,\ldots,n}(k|k), \boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{p}}_{1,2,\ldots,n}(k|k)}\right) \tag{19}$$

which indicates that the state estimate at time step k has been updated by the parallel measurements from sensors $1, 2, \ldots, n$ $(1 \leqslant n \leqslant N)$, namely

$$\hat{\boldsymbol{y}}^{\mathrm{p}}_{1,2,\ldots,n}(k|k) = \boldsymbol{y}(k|k-1) + \sum_{i=1}^{n} \boldsymbol{H}_i^{\mathrm{T}} \boldsymbol{R}_i^{-1} \hat{\boldsymbol{z}}_i(k) \tag{20}$$

$$\boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{p}}_{1,2,\ldots,n}(k|k)} = \left(p_{\mathrm{p}}(k) + \sum_{i=1}^{n} p_i(k)\right) \cdot \left(p_{\mathrm{p}}^{-1}(k) \boldsymbol{S}_{\mathcal{Y}(k|k-1)} + \sum_{i=1}^{n} p_i^{-1}(k) \boldsymbol{S}_{\mathcal{I}_i(k)}\right) \tag{21}$$

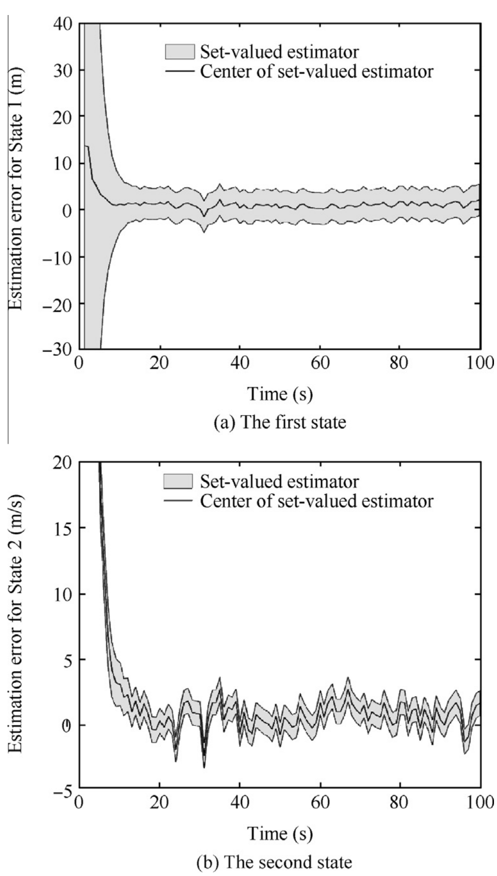

Fig. 1 Center of set of estimation errors and bounds for the first and second states in set-membership information filter.

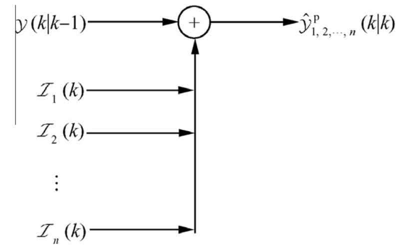

Fig. 2 Parallel set-membership information filter.

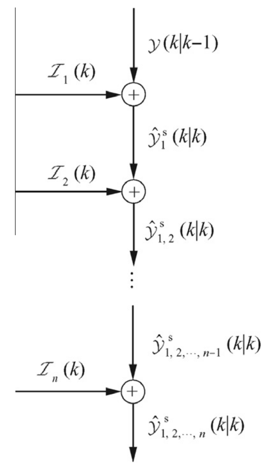

Fig. 3 Sequential set-membership information filter.

where p_{p} and p_i are given in Eq. (A16). In the following, we present the result of the equivalence analysis.

Theorem 2. *Let*

$$\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k) \oplus \mathcal{I}_n(k) \subseteq \hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k) \tag{22}$$

where

$$\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k) \triangleq \varepsilon\left(\hat{\boldsymbol{y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k), \boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k)}\right) \tag{23}$$

$$\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k) \triangleq \varepsilon\left(\hat{\boldsymbol{y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k), \boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k)}\right) \tag{24}$$

According to Eqs. (16) and (17), we have

$$\hat{\boldsymbol{y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k) = \hat{\boldsymbol{y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k) + \boldsymbol{H}_n^{\mathrm{T}} \boldsymbol{R}_n^{-1} \hat{\boldsymbol{z}}_n(k) \tag{25}$$

$$\boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k)} = \left(p_{1,2,\ldots,n-1}(k) + p_n(k)\right) \cdot \left(p^{-1}_{1,2,\ldots,n-1}(k) \boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k)} + p_n^{-1}(k) \boldsymbol{S}_{\mathcal{I}_n(k)}\right) \tag{26}$$

Then the following equations hold:

$$\hat{\boldsymbol{y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k) = \hat{\boldsymbol{y}}^{\mathrm{p}}_{1,2,\ldots,n}(k|k) = \boldsymbol{y}(k|k-1) + \sum_{i=1}^{n} \boldsymbol{H}_i^{\mathrm{T}} \mathbf{R}_i^{-1} \hat{\boldsymbol{z}}_i(k) \tag{27}$$

and

$$\boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k)} = \boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{p}}_{1,2,\ldots,n}(k|k)} = \left(p_{\mathrm{p}}(k) + \sum_{i=1}^{n} p_i(k)\right) \times \left(p_{\mathrm{p}}^{-1}(k) \boldsymbol{S}_{\mathcal{Y}(k|k-1)} + \sum_{i=1}^{n} p_i^{-1}(k) \boldsymbol{S}_{\mathcal{I}_i(k)}\right) \tag{28}$$

Appendix B shows the proof of Theorem 2.

5. Application to event-triggered cooperative target tracking in wireless sensor networks

In this section, we present how the results introduced above can be applied to the problem of event-triggered cooperative target tracking in WSNs. We consider a scenario with one target, ten sensors and a remote FC. A 1D constant velocity model for the target is given as

$$\begin{bmatrix} x(k) \\ \dot{x}(k) \end{bmatrix} = \begin{bmatrix} 1 & \mathrm{d}T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x(k-1) \\ \dot{x}(k-1) \end{bmatrix} + \boldsymbol{w}(k) \tag{29}$$

where x is the target position; $\mathrm{d}T = 1$ s is the step size of the discretization; and $\boldsymbol{w}$ is the Gaussian white noise with $\boldsymbol{Q}_c = \mathrm{diag}(1, 1)$. We assume that the sensors are able to obtain the 1D position measurements of the target with a sampling interval of 1 s and the variances of the measurement noise are the same as $R_1 = R_2 = \cdots = R_N = R = 1\ \mathrm{m}^2$. Furthermore, at each time step, the sensor may or may not send its current measurement to the FC according to a decision variable $d_i(k)$ that is determined by the SOD[16] triggering condition with the following form:

$$d_i(k) = \begin{cases} 0 & \left(z_i(k) - z_i(k_{\text{previous}})\right)^2 \leqslant S_e \\ 1 & \left(z_i(k) - z_i(k_{\text{previous}})\right)^2 > S_e \end{cases} \tag{30}$$

where S_e represents the triggering threshold. In this manner, while $d_i(k) = 1$, the ith sensor forwards its current measurement $z_i(k)$ to the FC, otherwise the FC formulates a set-membership pseudo measurement[23] for the ith sensor as

$$\mathcal{Z}_i(k) = \varepsilon(\hat{z}_i(k), S_e) = \varepsilon\left(z_i(k_{\text{previous}}), S_e\right) \tag{31}$$

As a result, the information transmission from the sensors to the FC becomes event-based, namely the communication is triggered only if the discrepancy between the current measurement and the one last sent exceeds a tolerable predefined threshold, and thus the communication cost reduces significantly.

In the following, we use the classical information filter with intermittent observations (IF-IO) as a benchmark to evaluate the performance of the proposed cooperative tracking strategy based on set-membership information filter (IF-SM). We first compare the communication cost between IF-IO and the proposed IF-SM in terms of the communication rate. After that, we compare the estimation accuracy in terms of the mean squared error (MSE).

Table 1 shows the comparison of communication rate between the two estimators, where $C_{\mathrm{SM},i}$ and $C_{\mathrm{IO},i}$ represent the communication rates for IF-SM and IF-IO respectively. In the following, we examine the mean difference between the communication rates of IF-SM and IF-IO by Student's t-test.

Let $\Delta_i = C_{\mathrm{SM},i} - C_{\mathrm{IO},i}$ for $i = 1, 2, \ldots, n$ where $n = 11$ in this case. We further assume that $\Delta_i \sim N(a, \sigma)$ where a and σ are both unknown, and set up the hypothesis as follows:

Table 1 Comparison of communication rates between IF-SM and IF-IO.

S_e	$C_{\mathrm{SM},i}$	$C_{\mathrm{IO},i}$
0.1	0.980	0.978
0.5	0.910	0.909
1.0	0.870	0.873
1.5	0.848	0.850
2.0	0.825	0.821
2.5	0.795	0.796
3.0	0.777	0.776
3.5	0.765	0.766
4.0	0.750	0.749
4.5	0.729	0.731
5.0	0.716	0.715

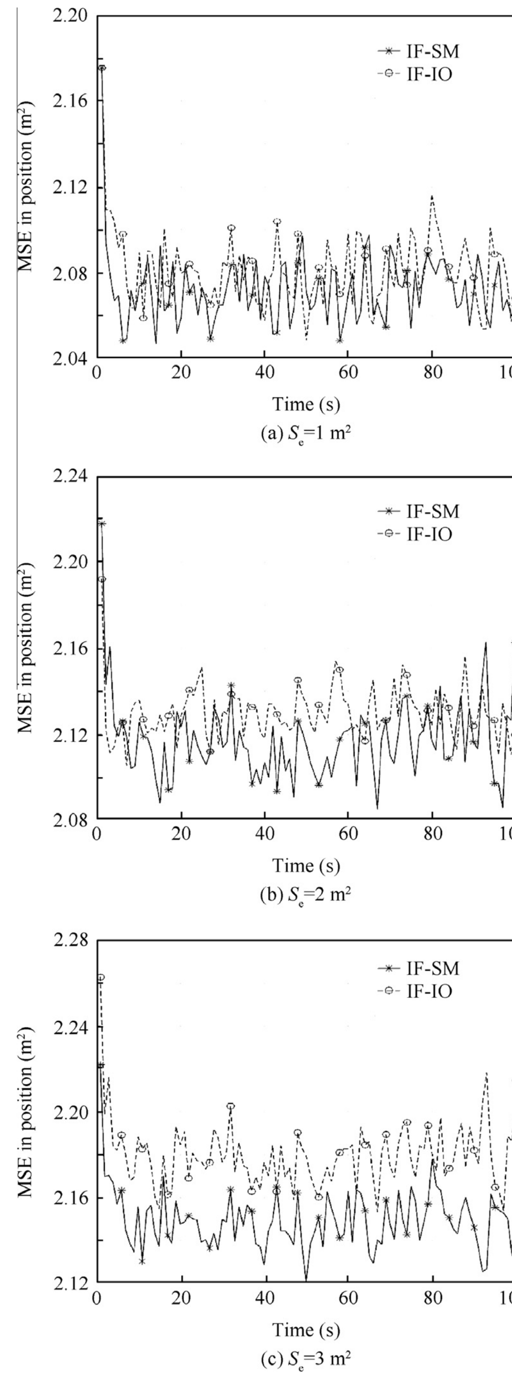

Fig. 4 Comparison of MSE in position between IF-SM and IF-IO with $S_e = 1, 2, 3\ \mathrm{m}^2$.

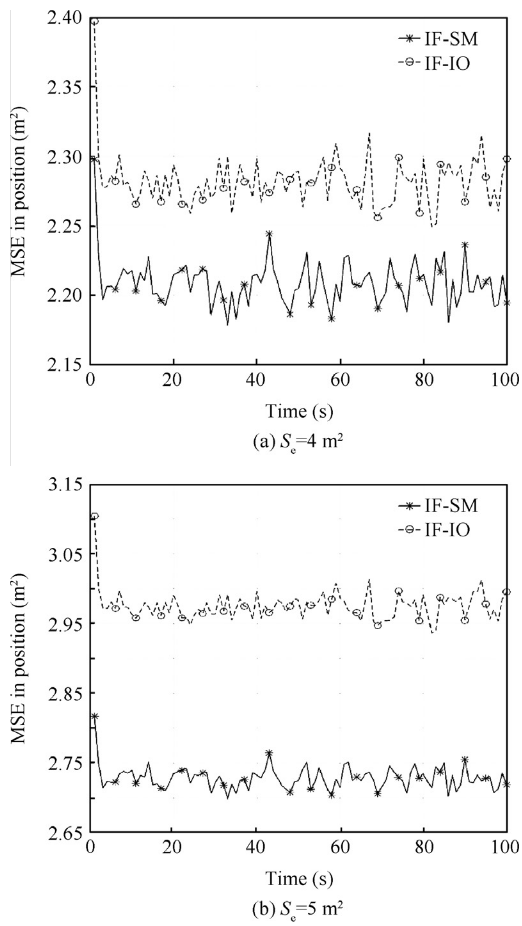

Fig. 5 Comparison of MSE in position between IF-SM and IF-IO with $S_e = 4, 5\ \mathrm{m}^2$.

$$H : a = a_0 = 0 \leftrightarrow K : a \neq a_0 \tag{32}$$

In this regard, the standard deviation S for the sample $\varDelta_i$ $(i = 1, 2, \ldots, n)$ is calculated as

$$S = \sqrt{\frac{\sum_{i=1}^{n}(\varDelta_i - \bar{\varDelta})^2}{n-1}} \tag{33}$$

where $\bar{\varDelta}$ is the sample mean, and the value of the one sample t-test is calculated as

$$T = \frac{\sqrt{n}(\bar{\varDelta} - a_0)}{S} = 0.1456 \tag{34}$$

Thus the hypothesis is accepted since $T < t_{n-1}\left(\frac{\alpha}{2}\right) = 1.812$ where $\alpha = 0.1$ and $t_{n-1}\left(\frac{\alpha}{2}\right)$ is the table value. As a result, the communication rates of IF-SM and IF-IO are close to each other as they apply the same event-triggered strategy SOD. Besides, it can be observed that increasing the triggering threshold S_e leads to the decrease of communication rate.

In the following, we compare the estimation accuracy in terms of the MSE. For the set-membership estimator, it is hard to distinguish which point of the set corresponds to the minimum estimation error at each step, since the actual state is always unknown. As a result, the center of the set of the estimation means is empirically regarded as an alternative point-valued estimation.[23,25,32] The results are obtained from a 1000-run Monte Carlo simulation.

Fig. 4 plots the MSE in position for IF-SM and IF-IO respectively, in which the performance of IF-SM is characterized by the center of the set of estimation means. It is observed that when the triggering threshold is relatively low ($S_e = 1,\ 2\ \mathrm{m}^2$), the tracking accuracy, namely the MSEs, of IF-SM and IF-IO is close to each other, since both of them operate in a high-rate communication environment. However, when the communication rate further decreases ($S_e \geqslant 3\ \mathrm{m}^2$), the impact of loss of track becomes more apparent, and in this case, all sensors might be simultaneously triggered off at a number of consecutive time steps. When such a condition happens, the MSE of IF-IO becomes higher than that of IF-SM for the reason that IF-IO is not available to correct the prior state estimation without new measurement. As a result, the discrepancy of MSE between IF-SM and IF-IO becomes larger with the increment of the triggering threshold S_e. For example, when $S_e = 3\ \mathrm{m}^2$, the MSE of IF-IO is only slightly larger than that of IF-SM (Fig. 4(c)). When $S_e = 4,\ 5\ \mathrm{m}^2$, the discrepancy of MSE between IF-IO and IF-SM keeps increasing (Fig. 5). The results demonstrate that the proposed event-triggered tracking strategy with IF-SM is robust against the change of triggering threshold.

6. Conclusions

This paper considers the problem of event-triggered cooperative target tracking in wireless sensor networks based on set-membership information filtering, where each sensor decides whether to send its current measurement or not according to the Send-on-Delta triggering mechanism. The major contribution of this paper lies in the study of some fundamental properties of the set-membership information filter, where we prove a sufficient condition for the asymptotic boundedness of the outer ellipsoidal approximation set of the estimation means. We also illustrate the equivalence between the parallel and sequential implementations of the set-membership information filter. Finally, we apply the set-membership information filter to a cooperative target tracking scenario. The simulation has proved the performance of the proposed method in the sense that it is robust against the change of triggering threshold.

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Appendix A

Proof of Theorem 1. Let

$$\mathcal{Y}(k|k-1) \triangleq \varepsilon(\boldsymbol{y}(k|k-1), \boldsymbol{S}_{\mathcal{Y}(k|k-1)}) \tag{A1}$$

Substitute Eqs. (13) and (A1) into Eq. (12), and we have

$$\mathcal{Y}(k|k) = \varepsilon(\boldsymbol{y}(k|k-1), \boldsymbol{S}_{\mathcal{Y}(k|k-1)}) \oplus \sum_{i=1}^{N} \varepsilon\left(\boldsymbol{H}_i^{\mathrm{T}} \boldsymbol{R}_i^{-1} \hat{\boldsymbol{z}}_i(k), \boldsymbol{S}_{\mathcal{I}_i(k)}\right) \tag{A2}$$

We assume that

$$\mathcal{Y}(k|k) \subseteq \hat{\mathcal{Y}}(k|k) \tag{A3}$$

where

$$\hat{\mathcal{Y}}(k|k) \triangleq \varepsilon\left(\hat{\boldsymbol{y}}(k|k), \boldsymbol{S}_{\hat{\mathcal{Y}}(k|k)}\right) \tag{A4}$$

Then according to Eqs. (16)–(18), we have

$$\hat{\boldsymbol{y}}(k|k) = \boldsymbol{y}(k|k-1) + \sum_{i=1}^{N} \boldsymbol{H}_i^{\mathrm{T}} \boldsymbol{R}_i^{-1} \hat{\boldsymbol{z}}_i(k) \tag{A5}$$

$$\begin{aligned} \boldsymbol{S}_{\hat{\mathcal{Y}}(k|k)} = &\left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) \\ &\cdot \left(p_{\mathrm{p}}^{-1}(k) \boldsymbol{S}_{\mathcal{Y}(k|k-1)} + \sum_{i=1}^{N} p_i^{-1}(k) \boldsymbol{S}_{\mathcal{I}_i(k)}\right) \end{aligned} \tag{A6}$$

where Eq. (A6) holds for $p_{\mathrm{p}} > 0$ and $p_i > 0$.

Since

$$\mathcal{Y}(k+1|k) = \boldsymbol{L}(k+1)\boldsymbol{F}^{-\mathrm{T}} \hat{\mathcal{Y}}(k|k) \tag{A7}$$

we further assume that

$$\mathcal{Y}(k+1|k) \triangleq \varepsilon(\boldsymbol{y}(k+1|k), \boldsymbol{S}_{\mathcal{Y}(k+1|k)}) \tag{A8}$$

and thus

$$\boldsymbol{y}(k+1|k) = \boldsymbol{L}(k+1)\boldsymbol{F}^{-\mathrm{T}} \hat{\boldsymbol{y}}(k|k) \tag{A9}$$

$$\boldsymbol{S}_{\mathcal{Y}(k+1|k)} = \boldsymbol{L}(k+1)\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\hat{\mathcal{Y}}(k|k)} \boldsymbol{F}^{-1} (\boldsymbol{L}(k+1))^{\mathrm{T}} \tag{A10}$$

Substitute Eq. (A6) into Eq. (A10), and then we have

$$\begin{aligned} \boldsymbol{S}_{\mathcal{Y}(k+1|k)} = &\boldsymbol{L}(k+1)\boldsymbol{F}^{-\mathrm{T}} \cdot \left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) \\ &\cdot \left(p_{\mathrm{p}}^{-1}(k) \boldsymbol{S}_{\mathcal{Y}(k|k-1)} + \sum_{i=1}^{N} p_i^{-1}(k) \boldsymbol{S}_{\mathcal{I}_i(k)}\right) \\ &\cdot \boldsymbol{F}^{-1} (\boldsymbol{L}(k+1))^{\mathrm{T}} \end{aligned} \tag{A11}$$

As a result, the objective is to investigate the boundedness of $\mathrm{tr}(\boldsymbol{S}_{\mathcal{Y}(k+1|k)})$. If $(\boldsymbol{F}, \boldsymbol{Q})$ is stabilizable and $(\boldsymbol{F}, \boldsymbol{H})$ is detectable, then $\boldsymbol{P}(k|k-1)$ converges towards the steady covariance $\bar{\boldsymbol{P}}_{\mathrm{prior}}$ as the solution to a Riccati equation[36] is as follows:

$$\bar{\boldsymbol{P}}_{\mathrm{prior}} = \boldsymbol{F}\bar{\boldsymbol{P}}_{\mathrm{prior}}\boldsymbol{F}^{\mathrm{T}} - \boldsymbol{F}\bar{\boldsymbol{P}}_{\mathrm{prior}}\boldsymbol{H}^{\mathrm{T}}\left(\boldsymbol{H}\bar{\boldsymbol{P}}_{\mathrm{prior}}\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{R}\right)^{-1}\boldsymbol{H}\bar{\boldsymbol{P}}_{\mathrm{prior}}\boldsymbol{F}^{\mathrm{T}} + \boldsymbol{Q} \tag{A12}$$

For steady state, we have

$$\begin{cases} \bar{\boldsymbol{S}} = \boldsymbol{H}\bar{\boldsymbol{P}}_{\mathrm{prior}}\boldsymbol{H}^{\mathrm{T}} + \boldsymbol{R} \\ \bar{\boldsymbol{K}} = \bar{\boldsymbol{P}}_{\mathrm{prior}}\boldsymbol{H}^{\mathrm{T}}\bar{\boldsymbol{S}}^{-1} \\ \bar{\boldsymbol{P}}_{\mathrm{posterior}} = (\boldsymbol{I} - \bar{\boldsymbol{K}}\boldsymbol{H})\bar{\boldsymbol{P}}_{\mathrm{prior}} \\ \bar{\boldsymbol{Y}} = \bar{\boldsymbol{P}}_{\mathrm{posterior}}^{-1} \\ \bar{\boldsymbol{M}} = \boldsymbol{F}^{-\mathrm{T}}\bar{\boldsymbol{Y}}\boldsymbol{F}^{-1} \\ \bar{\boldsymbol{C}} = \bar{\boldsymbol{M}}(\bar{\boldsymbol{M}} + \boldsymbol{Q}^{-1})^{-1} \\ \bar{\boldsymbol{L}} = \boldsymbol{I} - \bar{\boldsymbol{C}} \end{cases} \tag{A13}$$

Therefore Eq. (A11) can be rewritten as

$$\begin{aligned} \boldsymbol{S}_{\mathcal{Y}(k+1|k)} = &\bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}}\left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) \\ &\cdot \left(p_{\mathrm{p}}^{-1}(k) \boldsymbol{S}_{\mathcal{Y}(k|k-1)} + \sum_{i=1}^{N} p_i^{-1}(k) \boldsymbol{S}_{\mathcal{I}_i(k)}\right) \\ &\cdot \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}} \\ = &\left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) p_{\mathrm{p}}^{-1}(k) \bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\mathcal{Y}(k|k-1)} \\ &\cdot \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}} + \left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) \\ &\cdot \sum_{i=1}^{N} p_i^{-1}(k) \bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\mathcal{I}_i(k)} \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}} \end{aligned} \tag{A14}$$

Take trace on both sides of Eq. (A14),

$$\begin{aligned} \mathrm{tr}(\boldsymbol{S}_{\mathcal{Y}(k+1|k)}) = &\left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) p_{\mathrm{p}}^{-1}(k) \\ &\cdot \mathrm{tr}\left(\bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\mathcal{Y}(k|k-1)} \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}}\right) \\ &+ \left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) \\ &\cdot \sum_{i=1}^{N} p_i^{-1}(k) \mathrm{tr}\left(\bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\mathcal{I}_i(k)} \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}}\right) \end{aligned} \tag{A15}$$

and let

$$\begin{cases} p_{\mathrm{p}}(k) = \sqrt{\mathrm{tr}(\bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\mathcal{Y}(k|k-1)} \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}})} \\ p_i(k) = \sqrt{\mathrm{tr}(\bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\mathcal{I}_i(k)} \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}})} \end{cases} \tag{A16}$$

and then Eq. (A15) can be rewritten as

$$\begin{aligned} \mathrm{tr}(\boldsymbol{S}_{\mathcal{Y}(k+1|k)}) = &\left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) p_{\mathrm{p}}(k) \\ &+ \left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right) \sum_{i=1}^{N} p_i(k) \\ = &\left(p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k)\right)^2 \end{aligned} \tag{A17}$$

Hence,

$$\begin{aligned} \sqrt{\mathrm{tr}(\boldsymbol{S}_{\mathcal{Y}(k+1|k)})} = &p_{\mathrm{p}}(k) + \sum_{i=1}^{N} p_i(k) \\ = &\sqrt{\mathrm{tr}\left(\bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\mathcal{Y}(k|k-1)} \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}}\right)} \\ &+ \sum_{i=1}^{N} \sqrt{\mathrm{tr}\left(\bar{\boldsymbol{L}}\boldsymbol{F}^{-\mathrm{T}} \boldsymbol{S}_{\mathcal{I}_i(k)} \boldsymbol{F}^{-1}\bar{\boldsymbol{L}}^{\mathrm{T}}\right)} \\ = &\sqrt{\mathrm{tr}\left(\bar{\boldsymbol{A}} \boldsymbol{S}_{\mathcal{Y}(k|k-1)} \bar{\boldsymbol{A}}^{\mathrm{T}}\right)} + \sum_{i=1}^{N} \sqrt{\mathrm{tr}\left(\bar{\boldsymbol{A}} \boldsymbol{S}_{\mathcal{I}_i(k)} \bar{\boldsymbol{A}}^{\mathrm{T}}\right)} \\ = &\sqrt{\mathrm{tr}\left(\boldsymbol{S}_{\mathcal{Y}(k|k-1)} \bar{\boldsymbol{A}}^{\mathrm{T}} \bar{\boldsymbol{A}}\right)} + \sum_{i=1}^{N} \sqrt{\mathrm{tr}\left(\bar{\boldsymbol{A}} \boldsymbol{S}_{\mathcal{I}_i(k)} \bar{\boldsymbol{A}}^{\mathrm{T}}\right)} \end{aligned} \tag{A18}$$

From Ref.[37], it can be derived that if $\boldsymbol{S}_{\mathcal{Y}(k|k-1)} \geqslant \boldsymbol{0}$ and $\boldsymbol{0} \leqslant (\bar{\boldsymbol{A}})^{\mathrm{T}}\bar{\boldsymbol{A}} < \boldsymbol{I}$, then $\mathrm{tr}\left(\boldsymbol{S}_{\mathcal{Y}(k|k-1)}(\bar{\boldsymbol{A}})^{\mathrm{T}}\bar{\boldsymbol{A}}\right) \leqslant \lambda_{\max}\mathrm{tr}(\boldsymbol{S}_{\mathcal{Y}(k|k-1)})$ where $\lambda_{\max}$ is the largest eigenvalue of $(\bar{\boldsymbol{A}})^{\mathrm{T}}\bar{\boldsymbol{A}}$ and $0 \leqslant \lambda_{\max} \leqslant 1$. As a result, Eq. (A18) can be rewritten as

$$\sqrt{\mathrm{tr}\left(\boldsymbol{S}_{\mathcal{Y}(k+1|k)}\right)} \leqslant \sqrt{\lambda_{\max}}\sqrt{\mathrm{tr}\left(\boldsymbol{S}_{\mathcal{Y}(k|k-1)}\right)} + \sum_{i=1}^{N}\sqrt{\mathrm{tr}\left(\bar{\boldsymbol{A}}\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{R}_i^{-1}\boldsymbol{S}_{e_i(k)}\boldsymbol{R}_i^{-\mathrm{T}}\boldsymbol{H}_i\bar{\boldsymbol{A}}^{\mathrm{T}}\right)} \tag{A19}$$

which indicates that the set $\hat{\mathcal{Y}}$ is asymptotically bounded if all measurement sets $\mathcal{Z}_i$ are with bounded sizes $\mathrm{tr}(\boldsymbol{S}_{e_i(k)})$. At the same time, the size of the set $\mathcal{Y}$ is also bounded since $\hat{\mathcal{Y}}$ provides an outer approximation of $\mathcal{Y}$. □

Appendix B

Proof of Theorem 2. For $n = 1$, we have

$$\hat{\boldsymbol{y}}_1^{\mathrm{s}}(k|k) = \hat{\boldsymbol{y}}(k|k-1) + \boldsymbol{H}_1^{\mathrm{T}}\boldsymbol{R}_1^{-1}\hat{\boldsymbol{z}}_1(k) \tag{B1}$$

Then according to Eq. (25), we can derive that

$$\hat{\boldsymbol{y}}_{1,2,\ldots,n}^{\mathrm{s}}(k|k) = \hat{\boldsymbol{y}}(k|k-1) + \sum_{i=1}^{n}\boldsymbol{H}_i^{\mathrm{T}}\boldsymbol{R}_i^{-1}\hat{\boldsymbol{z}}_i(k) \tag{B2}$$

Therefore Eq. (27) holds.

According to Eq. (A16), the parameters in Eq. (26) should be given as

$$p_{1,2,\ldots,n-1}(k) = \sqrt{\mathrm{tr}\left(\boldsymbol{L}(k)\boldsymbol{F}^{-\mathrm{T}}\boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k)}\boldsymbol{F}^{-1}\boldsymbol{L}^{\mathrm{T}}(k)\right)} \tag{B3}$$

$$p_n(k) = \sqrt{\mathrm{tr}\left(\boldsymbol{L}(k)\boldsymbol{F}^{-\mathrm{T}}\boldsymbol{S}_{\mathcal{I}_n(k)}\boldsymbol{F}^{-1}\boldsymbol{L}^{\mathrm{T}}(k)\right)} \tag{B4}$$

We have

$$\begin{aligned}\boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k)} &= \boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{p}}_{1,2,\ldots,n-1}(k|k)} \\ &= \left(p_{\mathrm{p}}(k) + \sum_{i=1}^{n-1}p_i(k)\right) \\ &\quad \cdot \left(p_{\mathrm{p}}^{-1}(k)\boldsymbol{S}_{\mathcal{Y}(k|k-1)} + \sum_{i=1}^{n-1}p_i^{-1}(k)\boldsymbol{S}_{\mathcal{I}_i(k)}\right)\end{aligned} \tag{B5}$$

Substitute Eq. (B5) into Eq. (B3), and then we have

$$\begin{aligned}p_{1,2,\ldots,n-1}(k) &= \sqrt{\left(p_{\mathrm{p}}(k) + \sum_{i=1}^{n-1}p_i(k)\right)} \\ &\quad \cdot \sqrt{p_{\mathrm{p}}^{-1}(k)p_{\mathrm{p}}^2(k) + \sum_{i=1}^{n-1}p_i^{-1}(k)p_i^2(k)} \\ &= p_{\mathrm{p}}(k) + \sum_{i=1}^{n-1}p_i(k)\end{aligned} \tag{B6}$$

We further substitute Eqs. (B5) and (B6) into Eq. (26), and then we have

$$\begin{aligned}\boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n}(k|k)} &= \left(p_{1,2,\ldots,n-1}(k) + p_n(k)\right) \\ &\quad \cdot \left(p_{1,2,\ldots,n-1}^{-1}(k)\boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{s}}_{1,2,\ldots,n-1}(k|k)} + p_n^{-1}(k)\boldsymbol{S}_{\mathcal{I}_n(k)}\right) \\ &= \left(p_{\mathrm{p}}(k) + \sum_{i=1}^{n}p_i(k)\right) \\ &\quad \cdot \left(p_{\mathrm{p}}^{-1}(k)\boldsymbol{S}_{\mathcal{Y}(k|k-1)} + \sum_{i=1}^{n}p_i^{-1}(k)\boldsymbol{S}_{\mathcal{I}_i(k)}\right) \\ &= \boldsymbol{S}_{\hat{\mathcal{Y}}^{\mathrm{p}}_{1,2,\ldots,n}(k|k)}\end{aligned} \tag{B7}$$

Therefore Eq. (28) holds and the proof is completed. □

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